

$$\begin{aligned}
& + \frac{2}{r} \frac{\partial}{\partial z} \left( \frac{\partial \psi}{\partial r} - \frac{2\psi}{r} \right) \frac{\partial}{\partial z} \left( \frac{\bar{\sigma}}{\epsilon} \right) \\
& + 2 \frac{\partial}{\partial z} \left( 2 \frac{\partial \psi}{\partial r} - \frac{\psi}{r} \right) \frac{\partial^2}{\partial r \partial z} \left( \frac{\bar{\sigma}}{\epsilon} \right) = 0
\end{aligned} \tag{23}$$

where the operators  $\nabla_1$ ,  $\nabla_2$  and  $\nabla_3$  are defined as (these operators are equivalent to the standard Laplacian operator, except for the indicated sign changes).

$$\nabla_1^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

$$\nabla_2^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{\partial^2}{\partial z^2}$$

$$\nabla_3^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} - \frac{\partial^2}{\partial z^2}$$

Equation (23) represents the governing equation for determining the displacement function  $\psi$ , and is predicated on the existence of proportional straining, equation (4). If the total derivatives had been retained in the flow law equations, and if the velocities  $\dot{u}$  and  $\dot{w}$ , acting in the radial and axial directions, respectively, are defined as